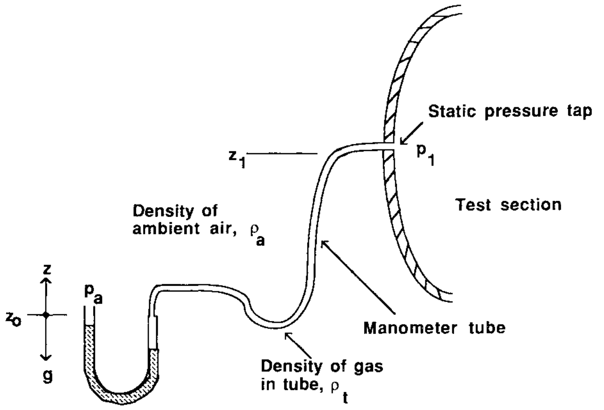

Fig. 1 Schematic diagram of typical pressure measurement setup.

low pressure differences, such as in a low-speed wind tunnel. A schematic diagram of a typical static pressure measurement setup is shown in Fig. 1. An electronic pressure transducer (or micromanometer) is often placed on a table at some elevation z_0, and the pressure tap is located at a different elevation z_1. The usual assumption is that the density ρ_t of the air inside the manometer tubing is the same as that of the ambient air ρ_a. In such a case, elevation difference $z_1 - z_0$ is irrelevant since the hydrostatic pressure rise in the atmosphere from z_1 to z_0 is identical to that in the tubing.

Observations

In our laboratory, while preparing for some high-accuracy pressure measurements with a setup similar to that of Fig. 1, it was discovered that the transducer zero was quite sensitive to elevation difference $z_1 - z_0$, contrary to intuition. Further investigation revealed that the air trapped inside the tubing was *denser* than the ambient air ($\rho_t > \rho_a$). Our analysis indicated a relative density difference of $(\rho_t - \rho_a)/\rho_a \approx 0.25$. In other words, the density of the gas inside the tubing was, in our test case, 25% larger than that of the ambient air! For a typical laboratory elevation difference of around 1 m, the corresponding pressure measurement error would be approximately 3.1 Pa (0.00045 psi, 0.012 in. of water.) Although this may at first appear negligible, note that the difference between stagnation and static pressure for the flow of air at 2.0 m/s is only about 2.4 Pa. The error due to the denser air in the tubing is certainly not negligible for low-speed flow measurements! Even at 10 m/s, if one tube from a pitot-static probe contained air at normal density, and the other tube contained air that was 25% more dense, the velocity reading from a transducer 1 m below the test section would read 10.25 m/s, an error of 2.5%.

Discussion

The obvious question is, How could the air inside the manometer tubing be so much denser than ambient air? The tubing used in our experiment was brand new, straight from its reel. Apparently, since the tubing is wound up like a garden hose immediately after it is manufactured, the gas inside the tubing would have virtually no chance to escape since the time of manufacture. The large observed density difference may be due to a number of factors, including outgassing of the plastic inside the tube or remnant gas from the manufacturing process. The authors have not analyzed the chemical composition of the gas trapped inside the manometer tubing. However, discussions with the manufacturer of the tubing (Hygenic Corporation of Ohio) verified our speculation that many types of plastic tubing are never blown out and may contain trapped gases such as water vapor and hydrocarbons. The concentrations of these gases are apparently high enough to increase the density of the air by about 25%.

In our laboratory, the situation was remedied by blowing out each section of manometer tubing with a small hand pump. The dense gas in the manometer tubing was thus replaced by fresh air, and the errors were completely eliminated. This is recommended by the authors as a preventative step prior to any pressure measurements, whenever new manometer tubing is being used.

Analytical Evaluation of Lattice Space Structures for Accuracy

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Introduction

EVALUATION of space structures has been performed conventionally in terms of structural strength, stiffness, vibrational characteristics, and controllability. However, the accuracy requirements for future large lattice space structures may require active control or static adjustment of the deformation. For example, a reflector support structure of a large space antenna will require extremely high accuracy to comply with electromagnetic requirements, the adaptive structures[1] need to be estimated for structural accuracy in order to design their controller systems, and a statically determinate structure also has an advantage of being free from thermal stresses.[2,3] To estimate the effects of random member length errors on the structural accuracy, some studies were reported earlier that were based on the approximate continuum analysis of the structures[4] and on multiple deterministic structural analyses of the truss (the Monte Carlo method) using the finite element analysis.[5]

The purpose of this study is to formulate the arbitrary stochastic effects of member length errors on the structural accuracy analytically. The comprehensive formulation presented here is based on vector subspaces associated with the dominant matrix, which is derived from the equilibrium matrix and the covariance matrix of the given structure, and it is shown that the stochastic analysis of the lattice-type structures can be treated as a simple eigenvalue problem, and that the invariant of the dominant matrix plays an important role in estimating the structural errors.

Effects of Random Member Length Errors

The errors induced in the lattice structures can be broadly divided into two types: stochastic and deterministic. The stochastic error is induced mainly due to member length tolerances, thermal expansions, and mission environment, whereas the deterministic error is mainly due to the nonlinearity of the various joints and the joint offsets. The present analysis regards that the errors can be represented by member length tolerances. Thus, this Note concentrates on the situation in which the nodal displacements are caused by errors in the lengths of the lattice members.

A three-dimensional lattice structure is defined as an assembly of straight members jointed at various nodal points. The relationship between the member length and the nodal position can be written as follows:

$$l_i^2 = (x_{i_1} - x_{i_2})^2 + (y_{i_1} - y_{i_2})^2 + (z_{i_1} - z_{i_2})^2 \tag{1}$$

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where l_i $(i=1,2,\ldots,m)$ is the ith member length and $(x_k, y_k, z_k)^T$ $(k=1,2,\ldots,j)$ is the kth node position. Thus the relationship between the member deformation vector $\mathrm{d}\boldsymbol{l}$ and the corresponding nodal displacements vector $\mathrm{d}\boldsymbol{x}$ can be written by the following in the matrix form,

$$\mathrm{d}\boldsymbol{l} = [A]\,\mathrm{d}\boldsymbol{x} \tag{2}$$

The equations of equilibrium at the nodes can be derived from the variational principle. The strain energy can be derived by using the deformation vector $\mathrm{d}\boldsymbol{l}$ and the member length error vector $\mathrm{d}\boldsymbol{s}$ as given in Eq. (3):

$$V = (½)(\mathrm{d}\boldsymbol{l} - \mathrm{d}\boldsymbol{s})^T[K](\mathrm{d}\boldsymbol{l} - \mathrm{d}\boldsymbol{s}) \tag{3}$$

where

$$[K] = \mathrm{diag}(E_iA_i/l_i), \qquad (i=1,2,\ldots,m) \tag{4}$$

Therefore, the equations of equilibrium at the nodes can be obtained as follows:

$$\big([A]^T[K][A]\big)\,\mathrm{d}\boldsymbol{x} = [A]^T[K]\,\mathrm{d}\boldsymbol{s} \tag{5}$$

When the lattice structure has some fixed nodes, the matrix $([A]^T[K][A])$ becomes a full rank matrix. Therefore, the nodal displacement vector is derived as

$$\mathrm{d}\boldsymbol{x} = \big([A]^T[K][A]\big)^{-1}[A]^T[K]\,\mathrm{d}\boldsymbol{s} \tag{6}$$

When the lattice structure has free-free boundary conditions, the rank of the matrix $([A]^T[K][A])$ becomes $(3j-6)$ because the lattice structure under a free-free boundary has the rigid mode nodal displacement. Therefore, in this case, the following two additional equations are required to suppress the rigid mode deformation:

$$\sum_{i=1}^{m}\int_{v_i}\rho_i\,\mathrm{d}\boldsymbol{x}_i\,\mathrm{d}v_i = \boldsymbol{0} \tag{7}$$

$$\sum_{i=1}^{m}\int_{v_i}\rho_i\boldsymbol{x}_i\times\mathrm{d}\boldsymbol{x}_i\,\mathrm{d}v_i = \boldsymbol{0} \tag{8}$$

where ρ_i is the material density and v_i indicates the volume of the ith member.

By using Eqs. (5), (7), and (8), the nodal displacement vector can be expressed as follows,

$$\mathrm{d}\boldsymbol{x} = [A']\,\mathrm{d}\boldsymbol{s} \tag{9}$$

In the special case where the lattice structure is statically determinate, the matrix $[A]$ becomes nonsingular; therefore, the nodal deviations depend only on the geometrical configurations of the lattice structure.

Analytical Procedure for Accuracy Estimation

There are some methods to estimate structural accuracy, such as maximum nodal displacement and variance of nodal displacements. In this Note, the variance of nodal displacements is considered.

The expected value of the errors is assumed to be zero. Thus, the covariance matrix of the member length error vector is obtained as given in Eq. (10):

$$E[\mathrm{d}\boldsymbol{s}\,\mathrm{d}\boldsymbol{s}^T] = [S]^2 \tag{10}$$

where $E[\ ]$ is an expected value function. When the errors are independent of each other, $[S]$ becomes a diagonal matrix consisting of a standard variation of truss members.

The arbitrary reference vector, such as position or directional vectors, can be considered by using an appropriate linear transformation:

$$\mathrm{d}\boldsymbol{x}^* = [A^*]\,\mathrm{d}\boldsymbol{s} \tag{11}$$

where $\mathrm{d}\boldsymbol{x}^*$ is a reference vector that has q elements, and $[A^*]$ is a corresponding matrix obtained from Eq. (9). From Eq. (11), the covariance matrix of the vector $\mathrm{d}\boldsymbol{x}^*$ is given as follows,

$$E[\mathrm{d}\boldsymbol{x}^*\,\mathrm{d}\boldsymbol{x}^{*T}] = [A^*][S]^2[A^*]^T \tag{12}$$

Now, the member length error vector $\mathrm{d}\boldsymbol{s}^*$ in the nondimensional form can be defined as follows,

$$\mathrm{d}\boldsymbol{x}^* = [A^*][S]\,\mathrm{d}\boldsymbol{s}^* \tag{13}$$

and the matrix $[A^*][S]$ dominates the characteristics of the structural error. The covariance matrix of the vector $\mathrm{d}\boldsymbol{s}^*$ is given as

$$E[\mathrm{d}\boldsymbol{s}^*\,\mathrm{d}\boldsymbol{s}^{*T}] = [I_m] \tag{14}$$

To consider the effects of member length errors on the reference vector $\mathrm{d}\boldsymbol{x}^*$, the length of the vector defined by Eq. (13) is used as shown in Eq. (15):

$$\mathrm{d}\boldsymbol{x}^{*T}\mathrm{d}\boldsymbol{x}^* = \mathrm{d}\boldsymbol{s}^{*T}[S][A^*]^T[A^*][S]\,\mathrm{d}\boldsymbol{s}^* \tag{15}$$

The matrix $([A^*][S])$ can be transformed as follows,

$$[A^*][S] = [T_1][\Lambda][T_2]^T \tag{16}$$

where

$$[\Lambda] = [\lambda_i\delta_{ij}], \qquad \big[\delta_{ij}=1\ (i=j);\quad \delta_{ij}=0\ (i\neq j)\big] \tag{17}$$

and $[T_1]$, which is a $(q\times q)$ matrix, and $[T_2]$, which is a $(m\times m)$ matrix, are both the orthonormal matrices, and the diagonal components of the matrix $[\Lambda]$ are the square root of the eigenvalue of the matrix $([S][A^*]^T[A^*][S])$. From Eq. (16), the following equation is derived,

$$[S][A^*]^T[A^*][S] = [T_2][\Lambda]^T[\Lambda][T_2]^T \tag{18}$$

When the following two transformations given in Eqs. (19) and (20) are considered,

$$\mathrm{d}\boldsymbol{u} = [T_2]^T\,\mathrm{d}\boldsymbol{s}^* \tag{19}$$

$$\boldsymbol{\sigma} = [\Lambda]\,\mathrm{d}\boldsymbol{u} \tag{20}$$

the covariance matrix of the vector $\mathrm{d}\boldsymbol{u}$ becomes a unit matrix, and the length of the vector $\boldsymbol{\sigma}$ is equal to that of the vector $\mathrm{d}\boldsymbol{x}^*$. From these results, $[T_2]$ transforms the member length error vector and the nodal displacement vector $\mathrm{d}\boldsymbol{x}^*$ to the eigensubspace. Therefore, the vectors $\mathrm{d}\boldsymbol{u}$ and $\boldsymbol{\sigma}$ can be considered to be equivalent of vectors $\mathrm{d}\boldsymbol{s}^*$ and $\mathrm{d}\boldsymbol{x}^*$, respectively. Thus, the covariance matrix of the vector $\boldsymbol{\sigma}$ is calculated as follows,

$$E[\boldsymbol{\sigma}\boldsymbol{\sigma}^T] = [\Lambda][\Lambda]^T \tag{21}$$

The expected value of the length of $\boldsymbol{\sigma}$ is obtained as shown in Eq. (22):

$$E[\boldsymbol{\sigma}^T\boldsymbol{\sigma}] = \mathrm{tr}\big([\Lambda]^T[\Lambda]\big) = \mathrm{tr}\big([\Lambda][\Lambda]^T\big) \tag{22}$$

The deformation shape of the structure can be expressed by the column vector of $[T_2]$.

From these results, the invariant of the dominant matrix $([S][A^*]^T[A^*][S])$ represents the structural errors; therefore, the variance of the reference vector can be directly calculated.

The formulations for the evaluation of structural accuracy can be adaptable to several types of errors. In the case of antenna reflectors, the surface error is evaluated by the rms error of surface deformations as follows,

$$\epsilon^2 = (1/A_s)\int_S w^2 \, dA_s \tag{23}$$

where w is the displacement from the best fit parabola surface. By using Eq. (15), the expected value of surface rms error can be obtained as shown in Eq. (24):

$$E[\epsilon^2] = \mathrm{tr}\big([\Lambda]^T[W][\Lambda]\big) = \mathrm{tr}\big([W][\Lambda][\Lambda]^T\big) \tag{24}$$

where $[W]$ is a weight matrix characterized by the configuration.

Application to Space Structures

In order to establish the feasibility of the present analytical investigation, a numerical example has been carried out, and in this example, a tetrahedral truss antenna reflector has been considered. In the analysis, a quantity ϵ_i is a nondimensionalized member length error ds_i with the corresponding member length l_i, and the variation of it is assumed to be equal and independent of each other as follows,

$$ds_i = \epsilon_i l_i, \qquad (i = 1,2,3,\ldots,m) \tag{25}$$

$$\sigma_\epsilon^2 = E[\epsilon_i^2] = \text{const} \tag{26}$$

The axial stiffness and the material density of all of the members are assumed to have equal values.

The tetrahedral trusses are found to be an attractive structural concept for antenna reflectors for meeting better surface accuracy requirements because of their high specific stiffness properties over the other various types. In order to demonstrate the present analytical procedures, a statically indeterminate tetrahedral truss antenna used by Greene[5] has been considered. The overall reflector has a hexagonal platform, which is characterized by a number of annular rings N_r. The reflector has a parabolic shape having focal length f and diameter D. At first, the geometry for the parabolic reflector is obtained by generating joint coordinates for the flat truss based on the number of rings and member lengths.

Figure 1 shows the effect of the number of subdivisions in the truss on the surface errors; these results are for truss reflectors with $f/D = 1.0$. The results of Ref. 4, which are shown as a horizontal line in this figure, are based on a continuum analysis, and the results of Ref. 5, indicated by the symbol ○,

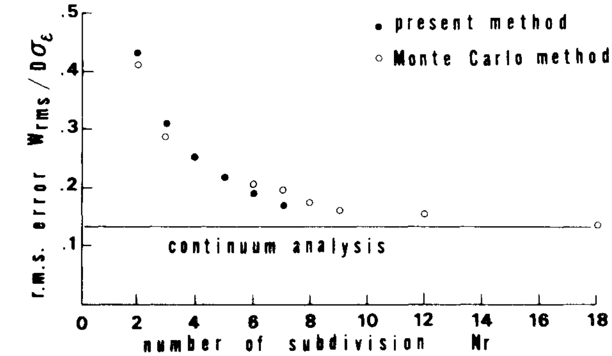

Fig. 1 Surface errors of tetrahedral antenna.

are calculated by the Monte Carlo Method using the finite element method. The results obtained by the present method (marked •) are found to be in good agreement with those of the finite element method.[5] Since the number of subdivisions increases, these results are close to the results obtained by a continuum model.

Concluding Remarks

The stochastic effects of member length errors on the accuracy of lattice space structures have been formulated analytically. The present formulation includes the effects of an arbitrary covariance among each structural element error. By this formulation, it has been shown that the invariant of the dominant matrix represents the variance of the structural errors and the stochastic analysis of the lattice-type structures can be treated as a simple eigenvalue problem. Therefore, these formulations enable a designer to assess analytically the potential of lattice structures for meeting accuracy requirements.

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